Information Sharing, Liquidity and Transaction Costs in Floor-Based Trading Systems.[1]

Thierry Foucault
HEC and CEPR
1, rue de la Libération
78351 Jouy en Josas, France.
Email:foucault@hec.fr

Laurence Lescourret
CREST and Doctorat HEC
15, Boulevard Gabriel Péri
92245 Malakoff, France.
Email:lescourr@ensae.fr

November, 2001

[1] We thank Giovanni Cespa, Asani Sarkar and seminar participants at CREST, Laval University, the AFFI2000 conference, the EEA 2000 conference, the FMA 2001 Meetings and the International Finance Conference Tunisie 2001. All errors are ours.

Abstract

Information Sharing, Liquidity and Transaction Costs in Floor-Based Trading Systems.

We consider information sharing between traders ("floor brokers") who possess different types of information, namely information on the payoff of a risky security or information on the volume of liquidity trading in this security. We interpret these traders as dual-capacity brokers on the floor of an exchange. We identify conditions under which the traders are better off sharing information. We also show that information sharing improves price discovery, reduces volatility and lowers expected trading costs. Information sharing can improve or impair the depth of the market, depending on the values of the parameters. Overall our analysis suggests that information sharing among floor brokers improves the performance of floor-based trading systems.

Keywords : Market Microstructure, Floor-Based Trading Systems, Open Outcry, Information Sharing, Information Sales.

JEL Classification Numbers : G10, D82.

1 Introduction

The organization of trading on the NYSE has been remarkably stable since its first constitution in 1817. Trading is conducted through open outcry of bids and offers of brokers acting on behalf of their clients or for their own account.[1] This trading mechanism is not unique to the NYSE. Equity markets like the Frankfurt Stock Exchange and the AMEX or derivatives markets like the CBOT and the CBOE are floor markets.[2] However floor-based trading mechanisms are endangered species as they are progressively replaced by fully automated trading systems[3]. Given this trend toward automation, it is natural to ask whether floor-based trading systems can provide greater liquidity and lower execution costs than automated trading systems. This question is of paramount importance for market organizers and traders. In fact, it has been hotly debated between members of Exchanges who considered switching from floor to electronic trading[4]. In order to survive floor-based trading mechanisms must outperform automated trading systems along some dimensions.

Automated trading systems dominate floor-based trading systems in many respects. First floor markets are more expensive to operate (see Domowitz and Steil (1999)). Second physical space limits the number of participants in floor markets but not in automated trading systems. Finally traders without an access to the floor are at an informational disadvantage compared with the traders on the floor. This disadvantage is likely to exacerbate agency problems between investors and their brokers (Sarkar and Wu (1999)).

By design, floor-based markets foster person-to-person contacts. Hence the ability of market participants to share information is greater in these markets. This feature is often viewed as being one advantage, if not the unique one, of floor-based trading systems.[5] For instance Harris (2000), p.8, points out that

[1] Of course, many trading rules have been changed since the creation of the NYSE. But it has always been a floor market. See Hasbrouck, Sofianos and Sosebee (1993) for a detailed description of the trading rules on the NYSE.

[2] In Frankfurt, the floor operates in parallel with an electronic trading system.

[3] The Marché à Terme International de France (MATIF), the Toronto Stock Exchange and The London International Financial Futures and Options Exchange (LIFFE) shut down their floor in 1997, 1998 and 2000, respectively.

[4] See the Economist (July 31st 1999): "A home grown revolutionary" and the Economist (August 26th 2000): "Out of the pits".

[5] Coval and Shumway (1998) show that the level of noise on the floor of CBOT's 30 year Treasury Bond futures affects price volatility. This also suggests that person to person contacts on the floor have an impact on price formation.

> *'Floor-based trading systems dominate electronic trading systems when brokers need to exchange information about their clients to arrange their trades.'*

Information sharing is a function of the floor which is difficult to replicate in electronic trading systems. These systems usually restrict the set of messages that can be sent by users (generally traders can only post prices and quantities). Furthermore trading in these systems is in most cases anonymous. This feature prevents traders from developing the reputation of honestly sharing information through enduring relationships.

Information sharing on the floor can take place between two types of participants. First floor-brokers can exchange information on their trading motivations with market-makers. Benveniste, Marcus and Whilelm (1992) model this type of information sharing and show that it mitigates adverse selection. Second floor-brokers can communicate with other floor-brokers. For instance, Sofianos and Werner (1997), p.6 notice that

> *'In addition, by standing in the crowd, floor brokers may learn about additional broker-represented liquidity that is not reflected in the specialist quotes: floor brokers will often exchange information on their intentions and capabilities, especially with competitors with whom they have good working relationships.'*

Our purpose in this paper is to analyze this type of information sharing. At first glance, information sharing among floor brokers is puzzling. In fact standard models with asymmetric information (e.g. Kyle (1985)) show that informed traders want to hide their information rather than disclose it to potential competitors. Furthermore, information sharing reinforces informational asymetries between those who share information and those who do not. It is therefore not obvious that it should improve market quality. Hence we address two questions. First, is it optimal for floor brokers to share information with their competitors? Second, what is the effect of information sharing among floor brokers on the overall performance of the market? In particular we study the impact of inter-floor brokers communication on standard measures of market quality, namely price volatility, price discovery, market liquidity and trading costs.

We model floor trading and information sharing using Kyle (1985)'s model as a workhorse. As in Roëll (1990), we assume that traders (floor brokers) have access to two types of information: (i) fundamental information which is information on the payoff of the security and (ii) non-fundamental information which is information on the volume of liquidity (non-informed) trading. We consider the possibility for two floor brokers endowed with different types of information (one has fundamental information and the other has non-fundamental information) to share information. More specifically we assume that floor brokers have information sharing agreements (they form a "clique"). An agreement specifies the precision with which each broker reports his or her information to the other broker. After receiving fundamental or non-fundamental information, the brokers in a clique pool their information according to the terms of their agreement just before submitting their orders for execution. We establish the following results.

- There is a wide range of parameters for which it is optimal for floor brokers to share their information (i.e. their expected profits are larger with information sharing).
- Information sharing can improve or impair the depth of the market, depending on the values of the parameters.
- Information sharing always reduces the *aggregate* trading costs for liquidity traders. However when information sharing impairs market depth, some liquidity traders are hurt.
- Information sharing occurs at the expense of the floor brokers who are not part to the information sharing agreement.
- Information sharing improves price discovery and reduces market volatility.

Intuitively information sharing intensifies competition between floor brokers and in this way it lowers the *total* expected profits of all floor brokers (reduces the aggregate trading costs). Information sharing also changes the allocation of trading profits among floor brokers. More specifically the floor brokers who share information capture a *larger* part of the total expected profits, at the expense of floor brokers who do not share information. These two effects explain why information sharing can simultaneously benefit liquidity traders and the floor brokers who share their information. Overall information sharing between floor brokers is an advantage for floor-based trading systems since it results in (a) lower trading costs, (b) faster price discovery and (c) lower price volatility. Interestingly, in line with our result, Venkataraman (2000) finds that trading costs on the NYSE are lower than on the Paris Bourse (an automated trading system), controlling for differences in stocks characteristics.[6]

[6] Theissen (1999) compares effective bid-ask spreads in an automated trading system (Xetra) and the floor of the Frankfurt Stock Exchange for stocks that trade in both systems. He finds that the average

Our analysis is related to the literature on information sales (e.g. Admati and Pfleiderer (1986), (1988) and Fishman and Hagerty (1995)). In contrast with this literature, we assume that the medium for information exchange is information, not money. Actually in our model, the trader who receives information rewards the information provider by disclosing another type of information. Hence we consider floor-based systems as markets for trading shares and forum to barter information. Another important difference is that we consider communication of information on the volume of liquidity trading. We show that it may be optimal to 'sell' (barter) such an information and that sales of non-fundamental information have an impact on market quality.

The model is described in the next section. Section 3 shows that it can be optimal for floor brokers to share information. Section 4 analyzes the impact of information sharing on various measures of market performance. Section 5 concludes. The proofs which do not appear in the text are in the Appendix.

2 The Model

2.1 Information Sharing Agreements

The Trading Crowd.

We consider a model of trading in the market for a risky security which is based on Kyle (1985). The final value of the security, which is denoted $\tilde{v}$, is normally distributed with mean μ and a variance σ_v^2 that we normalize to 1. This final value is publicly revealed at date 2. Trading in this security takes place at date 1. At this date, investors submit market orders to buy or to sell shares of the security. The excess demand (supply) is cleared at the price posted by a competitive and risk-neutral market maker.

The trading "crowd" for the security is composed of $N+1$ floor brokers.[7] At time 1, there are two types of floor brokers: (i) N fundamental speculators and (ii) one non fundamental speculator, B. Fundamental speculators have information on the final value of the security. For simplicity, as in Kyle (1985), we assume that they perfectly observe this final value, just before submitting their orders at date 1. Broker B, the non-fundamental

quoted spreads on the floor can be larger or smaller than in the automated trading system, depending on the stock characteristics. On average the quoted spreads are equal. This is consistent with our result that the impact of information sharing on market depth is ambiguous.

[7] The market-maker can also be considered as being a floor broker who has no information.

speculator, receives orders from liquidity traders. We denote $\tilde{x}_B$ the total quantity that broker B must execute on behalf of liquidity traders. As a whole, liquidity traders have a net demand equal to $\tilde{x} = \tilde{x}_0 + \tilde{x}_B$ shares. We assume that $\tilde{x}_0$ and $\tilde{x}_B$ are normally and independently distributed with means 0 and variances σ_0^2 and σ_B^2 respectively. We normalize the variance of the order flow due to liquidity trading, σ_x^2, to 1, i.e.:

$$\sigma_x^2 = \sigma_B^2 + \sigma_0^2 = 1.$$

In this way, σ_B^2 can be interpreted as broker B's market share of the total order flow from liquidity traders. The remaining part of the order flow can be seen as being intermediated by floor brokers who do not trade for their own account or as being routed electronically to the floor.[8],[9]

Both types of speculators can engage in proprietary trading. In particular broker B can act both as an agent (she channels a fraction of liquidity traders' orders) and as a principal (she submits orders for her own account). This practice is known as 'dual-trading' and is authorized in securities markets (see Chakravarty and Sarkar (2000) for a discussion).[10]Models with dual-trading include Röell (1990), Sarkar (1995) or Fishman and Longstaff (1992). In these models, as in the present article, brokers engaged in dual-trading exploit their ability to observe orders submitted by uninformed (liquidity) traders.[11] None of these models has considered information sharing of fundamental and non fundamental information among brokers, however. Our purpose is to study the effects of this activity. As argued in the introduction, this type of information exchange is a distinctive feature of floor markets. The speculators with fundamental information can be seen as brokers who exclusively trade for their own account (like scalpers and locals in derivatives markets). They could also be seen as brokers who have no customers' orders to execute at date 1.

It is reasonable to assume that the order flow from liquidity traders is independent across brokers (for instance brokers have different clients). In contrast, signals on the fundamental value of the security are correlated. For these reasons, we assumed that only one floor broker observes the non-fundamental information, $\tilde{x}_B$, whereas several floor

[8] In the US, full line brokerage houses engage in proprietary trading activities. Discount brokers do not, however.

[9] For instance, on the NYSE, orders can reach a market-maker through floor brokers or electronically through a system called SuperDot.

[10] For instance, Chakravarty and Sarkar (2000) observe that in the NYSE potential dual traders are national full line brokerage houses and the investment banks.

[11] See also Madrigal (1996). We borrow the distinction between 'fundamental' vs. 'non-fundamental' speculators from this author.

brokers observe the fundamental information, $\tilde{v}$. We have analyzed the model when there is more than one non-fundamental broker (with independent order flow) and brokers perfectly share information (information sharing is described below). The presentation of the model is more complex but the conclusions are qualitatively similar to those we obtain in the case with only one non-fundamental broker. One reason for which the model is more complex is that the number of cliques (groups of paired brokers with distinct information) is endogenous. In equilibrium, this number can be smaller than the maximum possible number of cliques. For instance if there is an equal number, N, of fundamental and non-fundamental brokers, the number of cliques can be smaller than N. In particular, with perfect information sharing, this is necessarily the case when $\sigma_0^2 = 0$. In this case, the aggregate order flow channeled by the non-fundamental brokers who are not affiliated to a clique plays the role of $\tilde{x}_0$ in the present article.

Information Sharing.

We model information sharing as follows. We assume that the non fundamental speculator, B, has an agreement to share information with one fundamental speculator, S. According to this agreement, before trading at date 1, the non-fundamental speculator sends a signal

$$\hat{x} = \tilde{x}_B + \tilde{\eta},$$

to the fundamental speculator. In exchange, the fundamental speculator sends a signal

$$\hat{v} = \tilde{v} + \tilde{\varepsilon},$$

to the non fundamental speculator. The random variables $\tilde{\eta}$ and $\tilde{\varepsilon}$ are independently and normally distributed with mean zero and variances σ_η^2 and σ_ε^2, respectively. We refer to the inverse of σ_η^2 (resp. σ_ε^2) as the *precision* of the signal sent by broker B (S). The larger is σ_η^2 (σ_ε^2), the less precise is the signal sent by speculator B (speculator S) and hence the lower is its informative value. Two polar cases are of particular interest. First there is *perfect information sharing* if $\sigma_\eta^2 = \sigma_\varepsilon^2 = 0$. Second there is *no information sharing* if $\sigma_\eta^2 = \sigma_\varepsilon^2 = 1$. In-between these two cases, there is information sharing but it is imperfect (at least one speculator does not perfectly disclose his or her information). The information sets of speculators B and S at date 1 are denoted $y_B = (\tilde{x}_B, \hat{x}, \hat{v})$ and $y_S = (\tilde{v}, \hat{x}, \hat{v})$, respectively.

In reality floor brokers are likely to exchange information with the brokers with whom they have enduring relationships. In this case their decision to share information with a given broker must be based on the long-term (average) benefits of information sharing. For this reason, we assume that the speculators decide to share information by comparing their ex-ante (i.e. prior to receiving information) expected profits with and without information sharing. We say that information sharing is possible if there exists a pair $(\sigma_\eta^2,\sigma_\varepsilon^2)$ such that the expected profits of speculator S *and* B are larger when there is information sharing. In section 3, we identify parameters' values for which information sharing is possible.

Remarks.

It is worth stressing that we focus on the possibility of an information sharing agreement but not on its implementation. In particular, we do not address enforcement issues. In that, we follow the literature on information sales where the quality of the information which is sold is assumed to be contractible.[12] We also assume that the information sharing agreement and its characteristics $(\sigma_\eta^2, \sigma_\varepsilon^2)$ are known by all participants (including the market-maker). This common knowledge assumption is also standard in the literature on information sales.

2.2 The equilibrium of the Floor Market

In this section, we derive the equilibrium of the trading stage at date 1, given the characteristics of the information sharing agreement between speculators B and S. Then, in the next section, we analyze whether or not it is optimal for B and S to exchange information.

We denote by $Q^S(y_S)$ and $Q^B(y_B)$, the orders submitted by speculators S and B, respectively. In the set of fundamental speculators, we assign index 1 to speculator S. An order submitted by the other fundamental speculators $i = 2, ..., N$ is denoted $Q^i(\tilde{v})$. The total excess demand that must be cleared by the competitive market maker is therefore

$$O = \sum_{i=2}^{i=N} Q^i(\tilde{v}) + Q^S(y_S) + Q^B(y_B) + \tilde{x}.$$

As the market maker is assumed to be competitive, he sets a price $p(O)$ equal to the asset

[12] See Admati and Pfleiderer (1986), (1988). Some papers have shown how incentives contracts can be used to induce an information provider to truthfully reveal the quality of his signal (see Allen (1990) or Bhattacharya and Pfleiderer (1985)). Reputation effects may also help to sustain information sharing agreements (see Benabou and Laroque (1992)).

expected value conditional on the net order flow, i.e.

$$p(O) = E(\tilde{v} \mid O). \tag{1}$$

An equilibrium consists of trading strategies $Q^S(.)$, $Q^B(.)$, $Q^i(.), i = 2, ..., N$ and a competitive price function $p(.)$ such that (i) each trader's trading strategy is a best response to other traders' strategies and (ii) the dealer's bidding strategy is given by Equation (1).[13] For given characteristics, $(\sigma_\eta^2, \sigma_\varepsilon^2)$, of an information sharing agreement, the next lemma describes the unique linear equilibrium of the trading game.

Lemma 1 *: The trading stage has a unique linear equilibrium which is given by*

$$\begin{aligned}
p(O) &= \mu + \lambda O, && (2)\\
Q^S(y_S) &= a_1(\tilde{v} - \mu) + a_2(\hat{v} - \mu) + a_3\hat{x}, && (3)\\
Q^i(\tilde{v}) &= a'(\tilde{v} - \mu), i = 2, ..., N && (4)\\
Q^B(y_B) &= b_1\tilde{x}_B + b_2\hat{x} + b_3(\hat{v} - \mu), && (5)
\end{aligned}$$

where coefficients $a_1, a_2, a_3, a', b_1, b_2, b_3$ *and* λ *are*

$$\begin{aligned}
a_1 &= \frac{3\left(\sigma_v^2 + \sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)},\\
a_2 &= -\frac{\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)},\\
a_3 &= -\frac{\sigma_B^2}{3\left(\sigma_B^2 + \sigma_\eta^2\right)},\\
a' &= \frac{2\sigma_v^2 + 3\sigma_\varepsilon^2}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)},\\
b_1 &= -\frac{1}{2},\\
b_2 &= \frac{\sigma_B^2}{6\left(\sigma_B^2 + \sigma_\eta^2\right)},\\
b_3 &= \frac{2\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)},
\end{aligned}$$

[13] More precisely, we consider the Perfect Bayesian Equilibria of the trading game.

and

$$\lambda(\sigma_\varepsilon^2, \sigma_\eta^2) = \frac{6\sqrt{\sigma_v^2\left(\sigma_B^2+\sigma_\eta^2\right)}\left(4\left(N+1\right)\sigma_v^4+\left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2+9N\sigma_\varepsilon^4\right)}{\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)\sqrt{\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)}}$$

Traders purchase (sell) the security when their estimation of the asset value is above (below) the unconditional expected value. Hence, the coefficients a_1, a' and b_3 are positive. Non fundamental information is also a source of profit. Intuitively liquidity traders' orders create temporary price pressures. Brokers with non-fundamental information are aware of these price pressures. They can profit from this knowledge by selling (buying) high (low) when liquidity traders buy (sell). More formally suppose that the fundamental speculators (but not the market maker) do not expect changes in the security value (i.e. $\tilde{v} = \mu$). Suppose also that B and S perfectly share information and that liquidity traders submit buy orders. These orders push the price upward because the market maker can not distinguish liquidity orders from informed orders. Speculators B and S however know that the correct value of the security is μ. In anticipation of the upward pressure on the clearing price, they submit sell orders. By symmetry, they submit buy orders when liquidity traders submit sell orders. This explains why coefficients b_1 and a_3 are negative. This means that floor brokers B and S partly accommodate liquidity traders' orders and reduce the order flow imbalance that must be executed by the market-maker. A similar effect is obtained in Röell (1990) and Sarkar (1995).

The previous discussion shows how speculators can profit both from fundamental and non fundamental information. Hence there is a benefit to exchange fundamental (non-fundamental) information for non-fundamental (fundamental) information. Information sharing is costly, however. Actually speculators S and B depreciate the value of their private information when they share it. Consider speculator B for instance. If she does not share information ($\sigma_\eta^2 = +\infty$), she accommodates half of the order flow she receives (since $b_1 = -1/2$). If she shares information then brokers B *and* S (instead of broker B alone) provide liquidity to the orders channeled by broker B. For instance if there is perfect information sharing then each broker accommodates one third of the orders received by broker B (since $b_1 + b_2 = -1/3$ and $a_3 = -1/3$ when $\sigma_\eta^2 = 0$). This competition for the provision of liquidity has two effects. First, broker B trades smaller quantities. Second, the order imbalance that must be executed by the market-maker is smaller. Hence, for a given price schedule (a fixed λ), prices react less to the order flow.[14] In fact speculator B reduces her trade size when she shares information (b_2 has a sign opposite the sign of b_1) precisely to mitigate this effect. These two effects (smaller trade size/smaller absolute price movements) reduce speculator's B profits on non-fundamental information. This is the cost of sharing non-fundamental information.

A similar argument holds for speculator S. He depreciates the value of fundamental information when he shares it with speculator B. In order to mitigate this effect, he adjusts his trading strategy to the message he sends to speculator B. This explains why a_2 has a sign opposite a_1.

To sum up, information sharing has benefits and costs. Information sharing is a source of profits since it allows each broker to trade on a new type of private information. But the brokers obtain new information only if they disclose all or part of their information. This is costly since it reduces the trading profits that can be made on the information originally possessed by a broker. In the next section we show that the benefit of information sharing can outweight its cost.

3 Is Information Sharing Possible?

In this section, we identify cases in which speculators B and S are better off when they share information. We start by considering the effect of the precisions with which the speculators B and S share their information on the market depth (measured by λ^{-1}).[15] It turns out that this effect is important to interpret the results.

Lemma 2 : *The depth of the market (i.e. λ^{-1}) is affected by the precisions with which the fundamental and the non fundamental speculators share their information.*

1. *The market depth increases with the precision of the signal sent by broker S ($\frac{\partial\lambda}{\partial\sigma_\varepsilon^2} > 0$),*
2. *The market depth decreases with the precision of the signal sent by broker B ($\frac{\partial\lambda}{\partial\sigma_\eta^2} < 0$).*

[14] In order to convey the intuition we take λ as given. However the slope of the price schedule is affected by information sharing. As shown below (Lemma 2) sharing non-fundamental information enlarges λ. This mitigates the loss in profit due to the second effect (smaller price changes).

[15] The market depth is the order flow necessary to change the price by 1 unit. The larger is the market depth, the greater is the liquidity of the market. Actually, when λ is small, the market-maker accommodates large order imbalances without substantial changes in prices.

Notice that an increase in the quality of the information provided by B to S enlarges λ, that is it decreases the depth of the market. The intuition for this result is as follows. Exchange of non-fundamental information increases the role of floor brokers (B and S) in the provision of liquidity. To see this point, let $Q^T = Q^B + Q^S$ be the total trade size of speculators B and S and consider their expected total trade size contingent on $\tilde{x}_B = x_B$. We obtain

$$E(Q^T \mid \tilde{x}_B = x_B) = (b_1 + b_2 + a_3)(x_B) = -\left(\frac{1}{2} + \frac{\sigma_B^2}{6(\sigma_B^2 + \sigma_\eta^2)}\right)(x_B). \tag{6}$$

The smaller is σ_η^2, the larger is the fraction ($|b_1+b_2+a_3|$) of the orders received by broker B which is accommodated by speculators S and B. As a consequence the dealer participates less to liquidity trades. In this sense the exchange of non-fundamental information 'siphons' uninformed order flow away from the market-maker. Thus this *siphon effect* increases his exposure to informed trading and the price schedule becomes steeper.[16]

Interestingly an increase in the quality of the information provided by S to B has exactly the opposite effect: it improves the depth of the market. In this case, the effect of information sharing is to increase competition among fundamental traders. Hence they scale back their order size (a_1 and a' decrease when σ_ε^2 decreases). This effect reduces the market-maker's exposure to informed trading and thereby makes the price schedule less steep.

We denote speculator j's ex-ante expected profit (i.e. before observing information) by $\Pi^j(\sigma_\eta^2, \sigma_\varepsilon^2, N)$. Using Lemma 1, we obtain the following result.

Lemma 3 *: For given values of σ_ε^2 and σ_η^2, the expected trading profits for speculators B*

[16] In equilibrium informed traders scale back their order size when λ increases. But this is insufficient to compensate the reduction in uninformed trading due to the siphon effect.

and S are

$$\Pi^S(\sigma_\eta^2,\sigma_\epsilon^2,N) = \left(\frac{\sigma_v^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(4\sigma_v^2+9\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)^2}+\frac{\lambda\sigma_B^4}{9\left(\sigma_B^2+\sigma_\eta^2\right)}\right)$$

$$\stackrel{def}{=} \Pi_f^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)+\Pi_{nf}^S(\sigma_\eta^2,\sigma_\varepsilon^2,N),$$

and,

$$\Pi^B(\sigma_\eta^2,\sigma_\varepsilon^2,N) = \left(\frac{4\sigma_v^4\left(\sigma_v^2+\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)^2}+\frac{\lambda\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)}{36\left(\sigma_B^2+\sigma_\eta^2\right)}\right)$$

$$\stackrel{def}{=} \Pi_f^B(\sigma_\eta^2,\sigma_\varepsilon^2,N)+\Pi_{nf}^B(\sigma_\eta^2,\sigma_\varepsilon^2,N).$$

Each speculator's expected profits has two components: (i) the expected profit she or he obtains by trading on fundamental information (Π_f^j) and (ii) the expected profit she or he obtains by trading on non-fundamental information (Π_{nf}^j). An information sharing agreement is viable if and only if both speculators B and S are better off when they share information. Hence an information sharing agreement is possible if and only if there exists a pair $(\sigma_\eta^2,\sigma_\varepsilon^2)$ such that

$$\Gamma_B\left(\sigma_\eta^2,\sigma_\varepsilon^2,N\right)\stackrel{def}{=}\Pi^B(\sigma_\eta^2,\sigma_\varepsilon^2,N)-\Pi^B(\infty,\infty,N)>0, \qquad (7)$$

and

$$\Gamma_S\left(\sigma_\eta^2,\sigma_\varepsilon^2,N\right)\stackrel{def}{=}\Pi^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)-\Pi^S(\infty,\infty,N)>0. \qquad (8)$$

The Γs' measure the expected surplus associated with the information sharing agreement for speculators B and S.

Proposition 1 *: The set of parameters for which speculators B and S share information is non-empty.*

We establish the result by providing 3 numerical examples. For each example, we report in Tables 1, 2 and 3 below the break-down of the trading profits for the different participants with and without information sharing. We also compare the market depth with and without information sharing. The examples have been chosen because they illustrate different phenomena that we will discuss in the rest of the paper. The trading profits are scaled by σ_v^2 and σ_x^2 that we normalize to 1 throughout the paper.

Proof:

Example 1: $\sigma_0^2 = 0$, $\sigma_\varepsilon^2 = 0$, $\sigma_\eta^2 = 2/3$, $N = 2$.

Profits and depth	**Information Sharing**	**No Information Sharing**
$(N - 1) \times \Pi_f^i \quad i \neq S, B$	0.0589	0.1178
Π_f^S	0.0589	0.1178
Π_{nf}^S	0.0707	0
Π_f^B	0.0589	0
Π_{nf}^B	0.1767	0.2357
Total Expected Profits	0.4242	0.4714
Market Depth (λ)	1.0607	0.9428

Table 1

In this case we obtain that

$$\Gamma_S = \Pi_f^S + \Pi_{nf}^S - \Pi^S(\infty, \infty) = 0.0589 + 0.0707 - 0.1178 = 0.0118,$$

and

$$\Gamma_B = \Pi_f^B + \Pi_{nf}^B - \Pi^B(\infty, \infty) = 0.0589 + 0.1767 - 0.2357 = 0.$$

Observe that the total surplus for speculators B and S is positive and equal to

$$\Gamma_S + \Gamma_B = 0.0118,$$

but that the total surplus for all speculators is negative and equal to

$$(N - 1)\Gamma_i + \Gamma_S + \Gamma_B = (0.0589 - 0.1178) + 0.0118 = -0.0471.$$

(Γ_i denotes the difference in the expected profit with and without information for a speculator different from S or B.)

Example 2: $\sigma_0^2 = 0.6$, $\sigma_\varepsilon^2 = 0$, $\sigma_\eta^2 = 0$, $N = 10$.

Profits and Depth	**Information Sharing**	**No Information Sharing**
$(N-1)\times\Pi_f^i \quad i \neq S, B$	0.1815	0.2165
Π_f^S	0.0202	0.0241
Π_{nf}^S	0.0153	0
Π_f^B	0.0202	0
Π_{nf}^B	0.0153	0.0344
Total Expected Profits	0.2525	0.2749
Market Depth (λ)	0.3443	0.3436

Table 2

In this case we obtain that

$$\Gamma_S = \Pi_f^S + \Pi_{nf}^S - \Pi^S(\infty, \infty) = 0.0114,$$

and

$$\Gamma_B = \Pi_f^B + \Pi_{nf}^B - \Pi^B(\infty, \infty) = 0.0011$$

Observe that the total surplus for speculators B and S is positive (0.0125) but that the total surplus for all speculators is negative (-0.0224.)

Example 3: $\sigma_0^2 = 0.6$, $\sigma_\varepsilon^2 = 0$, $\sigma_\eta^2 = 1.3$, $N = 10$.

Profits and Depth	**Information Sharing**	**No Information Sharing**
$(N-1)\times\Pi_f^i \quad i \neq s, B$	0.1874	0.2165
Π_f^S	0.0208	0.0241
Π_{nf}^S	0.0035	0
Π_f^B	0.0298	0
Π_{nf}^B	0.0290	0.0344
Total Expected Profits	0.2615	0.2749
Market Depth (λ)	0.033	0.034

Table 3

In this case we obtain that

$$\Gamma_S = \Pi_f^S + \Pi_{nf}^S - \Pi^S(\infty, \infty) = 0.0003,$$

and

$$\Gamma_B = \Pi_f^B + \Pi_{nf}^B - \Pi^B(\infty, \infty) = 0.0155$$

Observe that the total surplus for speculators B and S is positive and equal to $\Gamma_S + \Gamma_B = 0.0158$. The total surplus for all speculators is negative and equal to -0.0134. $\blacksquare$

In all the examples, the joint expected profits of speculators B and S increase when they share information. Notice that this is a necessary condition for information sharing. Actually Equations (7) and (8) imply that

$$\Pi^B(\sigma_\eta^2, \sigma_\varepsilon^2, N) + \Pi^S(\sigma_\eta^2, \sigma_\varepsilon^2, N) > \Pi^S(\infty, \infty, N) + \Pi^B(\infty, \infty, N).$$

At the same time, there is a decline in the joint expected profits of the speculators who do not share information. Eventually the total expected profits for all the speculators are lower in all the examples (this is always the case; see Proposition 5 in Section 4). In sum information sharing is a way for speculators B and S to secure a larger part of a smaller 'cake'. The fall in total profits is not surprising: information sharing increases competition between floor brokers. The surprising part is that the joint expected profits of speculators B and S can increase despite the decline in the total trading profits for the speculators. This is key since this is a necessary condition for information sharing. We now provide an explanation for this observation. The explanation is quite complex because several effects interplay.

Consider the following ratio

$$r_1(\sigma_\varepsilon^2, \sigma_\eta^2) \stackrel{def}{=} \frac{E(Q^T \mid \tilde{v} = v)}{E(Q^T \mid \tilde{v} = v, \sigma_\varepsilon^2 = \infty, \sigma_\eta^2 = \infty)}.$$

This ratio compares the expected total trade size (Q^T) of the clique formed by speculators B and S conditional on fundamental information with and without an information sharing agreement. Using Lemma 1, we can write this ratio as

$$r_1(\sigma_\varepsilon^2, \sigma_\eta^2) = \frac{a_1(\sigma_\varepsilon^2, \sigma_\eta^2) + a_2(\sigma_\varepsilon^2, \sigma_\eta^2) + b_3(\sigma_\varepsilon^2, \sigma_\eta^2)}{a_1(\infty, \infty)}.$$

Hence $r_1 > 1$ means that the clique formed by B and S trades more aggressively on fundamental information when there is information sharing than when there is not. Using the expressions for a_1, a_2 and b_3 given in Lemma 1, we eventually obtain

$$r_1(\sigma_\varepsilon^2, \sigma_\eta^2) = (\frac{\lambda(\infty, \infty)}{\lambda(\sigma_\epsilon^2, \sigma_\eta^2)})(\frac{(4\sigma_v^2 + 3\sigma_\epsilon^2)(N+1)}{2(N+2)\sigma_v^2 + 3(N+1)\sigma_\epsilon^2}).$$

As $\lambda(\infty, \infty) > \lambda(0, \infty)$ (Lemma 2), it is immediate that $r_1(0, \infty) > 1$. By continuity, this inequality also holds true for other values of σ_ϵ^2 and σ_η^2. Hence there exist information sharing agreements which induce the clique formed by B and S to trade more aggressively. In turn this forces speculators who are not part of the clique to shade their total trade size. To see this point consider the following ratio

$$r_2(\sigma_\epsilon^2, \sigma_\eta^2) \overset{def}{=} \frac{E((N-1)Q^i \mid \tilde{v} = v)}{E((N-1)Q^i \mid \tilde{v} = v, \sigma_\varepsilon^2 = \infty, \sigma_\eta^2 = \infty)} = (\frac{\lambda(\infty, \infty)}{\lambda(\sigma_\epsilon^2, \sigma_\eta^2)})(\frac{(2\sigma_v^2 + 3\sigma_\epsilon^2)(N+1)}{2(N+2)\sigma_v^2 + 3(N+1)\sigma_\epsilon^2}),$$

where $(N-1)Q^i$ is the total trade size of speculators different from B and S. Using Lemma 2, we deduce that r_2 increases with σ_η^2. This implies that

$$r_2(\sigma_\epsilon^2, \sigma_\eta^2) \leq r_2(\sigma_\epsilon^2, \infty).$$

Using the expressions for $\lambda(\infty, \infty)$ and $\lambda(\sigma_\epsilon^2, \infty)$ given in the proof of Lemma 2, we obtain[17]

$$r_2(\sigma_\epsilon^2, \infty) < 1 \quad \forall \sigma_\epsilon^2 < \infty.$$

We conclude that $r_2(\sigma_\epsilon^2, \sigma_\eta^2) < 1$. This means that information sharing agreements force the speculators who are not part of the clique to trade less aggressively on their information. Hence the speculators who share information appropriate a larger share of the total profits which derive from trading on fundamental information.[18] For this reason, information sharing enlarges their joint expected profit on fundamental information. This is the case for instance in Examples 2 and 3.

Now consider the effect of information sharing on the profits which derive from non-fundamental information. On the one hand, there are more speculators who accommodate the order flow brokered by B. This effect decreases the level of expected profit on non-fundamental information. On the other hand the exchange of non fundamental information

[17] The proof requires straightforward manipulations and is available upon request.

[18] Notice that speculators in our model are like Cournot competitors. In Cournot competition, each firm would like to commit to trade a larger size than it does in equilibrium. This commitment would force other firms to trade in smaller sizes. In this way the committed firm can capture a larger share of the total profits. Intuitively sharing fundamental information is a way to make this commitment credible. This effect has been pointed out by Fishman and Hagerty (1995) in a model of information sale.

decreases the market depth and this effect increases profits from non-fundamental speculation as can be seen from Lemma 3. It turns out that there are cases (for instance Example 1) in which the second effect dominates and the *joint* expected trading profits of speculators S and B on non-fundamental information are larger when there is information sharing or

$$\Pi^B_{nf}(\sigma^2_\eta, \sigma^2_\varepsilon, N) + \Pi^S_{nf}(\sigma^2_\eta, \sigma^2_\varepsilon, N) - \Pi^B_{nf}(\infty, \infty, N) \geq 0, \quad for \quad \sigma^2_\eta < \infty \quad and \quad \sigma^2_\varepsilon < \infty$$

Observe that this can occur only when information sharing impairs market depth (increases λ). In Example 3, information sharing improves market depth and the joint expected profit on non-fundamental information decreases.

To sum up, there are two reasons why information sharing can increase the joint expected profits of speculators B and S:

- Sharing fundamental information allows the coalition formed by brokers S and B to trade more aggressively on fundamental information and to capture thereby a larger share of the total profits from speculation on fundamental information.

- Sharing non-fundamental information can reduce the market depth. This implies that prices react more to order imbalances. Larger total expected profits from speculation on non-fundamental information follows.

The precisions with which the speculators share their information determine how the surplus $(\Gamma_S + \Gamma_B)$ created by information sharing is split between brokers B and S. For instance, consider Examples 2 and 3. The value of σ^2_η is larger in Example 3, but otherwise the values of the parameters are identical in the two examples. The surplus for speculator $B(S)$ is larger (lower) in Example 3 than in Example 2. In line with intuition, for a fixed value of σ^2_ε, speculator $B(S)$ prefers to provide (receive) an information of low (high) quality. Hence speculators B and S have conflicting views over the information sharing agreements which should be chosen. It is also worth stressing that the size of the surplus created by information sharing depends on the precisions with which traders share information. For instance the joint surplus is smaller in Example 2 than in Example 3. In this paper, we do not study how traders select the characteristics of their information sharing agreement (σ^2_ϵ and σ^2_η). This is not necessary because our statements regarding market performance (next section) only depends on the existence of information sharing agreements, not on the specific values chosen for σ^2_ε and σ^2_η.

We now consider in more details information sharing agreements in which speculators B and S perfectly share information ($\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$). Perfect information sharing is of interest because it is relatively easy to implement. Actually, if there is perfect information sharing, B knows which quantity S should trade and vice versa (in our model they optimally trade the same quantity). Consequently, one speculator can detect cheating by the other speculator by observing his or her trade size.

Proposition 2 *: For $N \geq 2$, there exist two cut-off values (i) $\underline{\sigma}_0^2(N)$ and (ii) $\sigma_0^{*2}(N)$ such that perfect information sharing is possible if and only if $\sigma_0^2 \in [\underline{\sigma}_0^2(N), \sigma_0^{*2}(N)]$. Furthermore the cutoff values increase with N and are such that $0 < \underline{\sigma}_0^2(N) < \sigma_0^{*2}(N) < 1$.*

The proposition shows that perfect information sharing is possible if broker B does not channel a too large or a too small fraction of the order flow from liquidity traders. Observe that profits made on non-fundamental information (Π_{nf}^j) are proportional to the amount of liquidity trading brokered by B ($\sigma_B^2 = 1 - \sigma_0^2$). Hence σ_0^2 determines the value of non-fundamental information. Perfect information sharing can take place when this value is neither too large, nor too small. If the value of non-fundamental information is large ($\sigma_0^2 < \underline{\sigma}_0^2(N)$), the cost of disclosing her information perfectly for B (smaller profits on non-fundamental information) is large compared to the benefit (the possibility to profit from fundamental information). In order to attenuate this cost, B must therefore send a noisy signal to S. When the value of non-fundamental information is small ($\sigma_0^2 > \sigma_0^{*2}(N)$), the benefit of perfect information sharing is small for the fundamental speculator. Therefore he refuses to perfectly disclose his information.

The larger is the number of fundamental speculators, the smaller must be the fraction of liquidity traders' order flow brokered by B to sustain a perfect information sharing agreement ($\underline{\sigma}_0^2(N)$ increases with N). Actually the profits from fundamental information decrease with the number of fundamental speculators. The value of fundamental information is therefore small when N is large. Hence broker B accepts to perfectly disclose her information only if the value of non-fundamental information is itself small. The last part of the proposition implies that for all values of N, there exist values of $\sigma_0^2 < 1$ such that a perfect information sharing agreement can be sustained. Figure 1 plots $\underline{\sigma}_0^2(N)$ and $\sigma_0^{*2}(N)$ for different values of $N \geq 2$ and shows when perfect information sharing is possible.[19]

[19] The cutoff values $\underline{\sigma}_0^2(N)$ and $\sigma_0^{*2}(N)$ are implicitly defined in the proof of Proposition 2.

Remark. In the model we assume that brokers' roles are fixed: one has fundamental information and the other has non-fundamental information. Another possibility is that the roles are randomly allocated before trading and unkwnown at the time brokers decide to share information. For simplicity, assume that each broker in the clique has an equal probability to be the broker endowed with non-fundamental information. In this case, brokers agree to share information iff

$$\Pi^B(\sigma_\eta^2, \sigma_\varepsilon^2, N) + \Pi^S(\sigma_\eta^2, \sigma_\varepsilon^2, N) > \Pi^S(\infty, \infty, N) + \Pi^B(\infty, \infty, N).$$

This condition is always satisfied when $(\sigma_\varepsilon^2, \sigma_\eta^2)$ are such that Conditions (7) and (8) are satisfied. Hence if an information sharing agreement is possible when brokers' roles are fixed, it is still possible when brokers' role are randomly chosen.

4 Information Sharing and Market Performance

In this section, we analyze the effects of information sharing on traditional measures of market quality: (1) the informational efficiency of prices (measured by $Var(\tilde{v} \mid p)$), (2) price volatility (measured by $Var(\tilde{v} - p)$), (3) market depth (measured by λ) and (4) the expected trading costs borne by liquidity traders (i.e. their expected losses, $E(\tilde{x}(p - \tilde{v}))$). These aspects of market performance play a prominent role in the debates regarding the design of trading systems and have attracted considerable attention in the literature (see Madhavan (1996) or Vives (1995) for instance).

Proposition 3 *: Prices are more informative ($Var(\tilde{v} \mid p)$ smaller) and less volatile ($Var(\tilde{v} - p)$ smaller) when there is information sharing.*

The intuition behind this result is simple. When speculators S and B share information, the number of speculators trading on fundamental information increases. It follows that the aggregate order flow is more informative. For this reason, prices are more accurate predictors of the final value of the security and price discovery is improved.

We now examine the impact of information sharing on the depth of the market. As shown by Lemma 2, an increase in the precision with which speculator S transmits his information improves market depth. However, an increase in the precision with which speculator B transmits her information impairs market depth (because of the siphon effect).

Hence the impact of information sharing on market depth can be positive or negative. Of course, for the parameters such that information sharing occurs, one effect could be dominant. However Examples 2 and 3 in the previous section show that this is not the case. In these examples, σ_ϵ^2 and σ_η^2 are such that (i) information sharing is optimal and (b) information sharing impairs market depth (Example 2) or improves market depth (Example 3). The next proposition considers the effect of perfect information sharing on market depth. To this end, we define

$$\bar{\sigma}_0^2(N) = \frac{1 - h^2(N)}{8h^2(N) - 3} < 1,$$

where $h(N) = \frac{2(N+2)\sqrt{N}}{3(N+1)\sqrt{N+1)}} < 1$.

Proposition 4 *: Perfect information sharing improves market depth if and only if $\sigma_0^2 \geq \bar{\sigma}_0^2(N)$.*

Hence perfect information sharing improves market depth when broker B receives a sufficiently small fraction of the total order flow ($\sigma_0 \geq \bar{\sigma}^2(N)$). Recall that when there is perfect information sharing, σ_0^2 must be larger than a threshold ($\underline{\sigma}_0^2(N)$). Figure 2 depicts $\bar{\sigma}^2(N)$ (dotted line) when N increases. As it can be seen, there are values of σ_0^2 and N such that perfect information sharing occurs and impairs market liquidity (all the values below the dotted line and above the plain line).[20]

Notice that the market depth is related to the bid-ask spread. Actually a buy order of size q pushes the price upward by λq whereas a sell order of the same size pushes the price downward by λq. Hence

$$s(q) = p(q) - p(-q) = 2\lambda q,$$

can be interpreted as the bid-ask spread for an order of size q in our model (see Madhavan (1996)). The spread increases with λ. Accordingly the impact of information sharing on bid-ask spreads is ambiguous. Interestingly empirical studies which compare bid-ask spreads in floor-based trading systems and automated trading systems have not found that spreads were systematically lower in one trading venue. For instance, several studies (Kofman and Moser (1997), Pirrong (1996) and Shyy and Lee (1995)) have compared the bid-ask spreads on LIFFE (when it was a floor market) and DTB (an automated

[20] For large values of N, the difference $(\bar{\sigma}_0^2(N) - \sigma_0^{*2}(N))$ becomes smaller and smaller but is never zero. That is even for N large, there are values for σ_0^2 such that perfect information sharing takes place and impairs market depth.

trading system) for the same security (namely the German Bund futures contract). Kofman and Moser (1997) find that spreads are equal in the two markets; Pirrong (1996) reports narrower spreads on DTB whereas Shyy and Lee (1995) find smaller spreads on LIFFE. In April 1997, the Toronto Stock Exchange closed its trading floor and introduced an electronic trading system. Griffiths et al. (1998) compare bid-ask spreads for stocks listed on the Toronto Stock Exchange before and after the switch to the automated trading system. They do not find significant changes in quoted spreads.

Finally we consider the effects of information sharing on the aggregate expected trading costs for the liquidity traders. These expected trading costs are

$$E(TC) = E(\tilde{x}(p - \tilde{v})) = \underbrace{E(\tilde{x}_B(p - \tilde{v}))}_{\textit{Orders channeled by } B} + \underbrace{E(\tilde{x}_0(p - \tilde{v}))}_{\textit{Orders not channeled by } B}.$$

In the last expression, we distinguish between the expected trading costs for the liquidity traders who send their orders to broker B and the expected trading costs for those who do not. Using Lemma 1, we obtain that

$$E(\tilde{x}_B(p - \tilde{v})) = E(\tilde{x}_B E(p - \tilde{v} \mid \tilde{x}_B = x_B)) = \lambda E(\tilde{x}_B^2(1+b_1+b_2+a_3)) = \lambda \left(\frac{2\sigma_B^2 + 3\sigma_\eta^2}{6\left(\sigma_B^2 + \sigma_\eta^2\right)} \right) \sigma_B^2,$$

and

$$E(\tilde{x}_0(p - \tilde{v})) = \lambda \sigma_0^2.$$

Hence we rewrite the expected trading costs as

$$E(TC) = \underbrace{\lambda g(\sigma_\eta^2)\sigma_B^2}_{\textit{Orders channeled by } B} + \lambda\sigma_0^2,$$

with $g(\sigma_\eta^2) = \left(\frac{2\sigma_B^2+3\sigma_\eta^2}{6\left(\sigma_B^2+\sigma_\eta^2\right)} \right)$. The ratio $g(\sigma_\eta^2)$ increases with σ_η^2. Hence when information sharing improves market depth, it also decreases the expected trading costs for **all** liquidity traders : (1) the liquidity traders whose orders are channeled through broker B and (2) the other liquidity traders. For instance, with perfect information sharing this occurs when $\sigma_0^2 \in [\bar{\sigma}_0^2(N), \sigma_0^{*2}(N)]$.

When information sharing impairs market depth (increases λ), the expected trading costs of the liquidity traders who do not send their order to broker B increase. However the expected trading costs for the liquidity traders who use B's services decline despite the decrease in market depth. Actually information sharing increases competition among traders providing counter-parties to B's clients. Therefore a smaller fraction of the orders submitted by B's clients must be executed against the market-maker when speculators S and B share non fundamental information (see Equation (6)). The next proposition shows that the reduction in the expected trading costs for B's clients always dominates the increase in expected trading costs for the other liquidity traders.

Proposition 5 : *The expected trading costs borne by the liquidity traders are always smaller when there is information sharing.*

The trading game is a zero-sum game in this model. This implies that the expected trading costs borne by liquidity traders are equal to the speculators aggregate expected profits. Let $\Pi^a(\sigma_\eta^2, \sigma_\varepsilon^2, N)$ be speculators' aggregate expected profits. We have

$$E(TC) = \Pi^a(\sigma_\eta^2, \sigma_\varepsilon^2, N) \stackrel{def}{=} \Pi^S + \Pi^B + (N - 1)\Pi^i,$$

where $\Pi^i(\sigma_\eta^2, \sigma_\varepsilon^2, N)$ is the expected profit of a speculator who is not part to the information sharing agreement. Recall that a necessary condition for information sharing is that it increases the joint expected profits of speculators B and S, i.e. $\Pi^S + \Pi^B$. Since information sharing decreases the aggregate expected profits of all speculators, it follows that the joint expected profit of speculators $i \in \{2, ..., N\}$ decreases. Therefore, the concomitant decrease in trading costs for liquidity traders and increase in total expected profits for speculators S and B occur at the expense of the speculators who do not share information. Observe that this cannot happen when there is a single fundamental speculator ($N = 1$). In fact in this case, it is possible to show that there are no values for the parameters for which information sharing is optimal for B and S.

Overall the results of this section show how information sharing on the floor can improve the quality of floor-based markets along several dimensions. Information sharing makes price more informative, less volatile and fosters competition between floor brokers, so that ultimately the aggregate trading costs borne by the traders without an access to the floor are lower.

5 Conclusion

In this paper we have analyzed pre-trade information sharing between two two traders endowed with different types of information, namely fundamental or non-fundamental information. We find that there are cases in which the two traders are better off sharing their information. Information sharing improves price discovery and decreases volatility. We also show that information sharing decreases the aggregate expected trading costs borne by liquidity traders. Finally the effect of information sharing on market depth and bid-ask spreads is ambiguous.

Floor-based trading systems are designed in such a way that they greatly facilitate information sharing among floor brokers. Overall our results show how this feature can improve their performance. An interesting question is whether the benefits brought up by information sharing are outweighted by inherent disadvantages of floor-based systems (such as lack of transparency or larger operating costs). This issue is left for future research.

References

[1] Admati A., and Pfleiderer P. (1986), "A Monopolistic Market for Information", *Journal of Economic Theory*, 39, 400–438.

[2] Admati A., and Pfleiderer P. (1988), "Selling and Trading on Information in Financial Markets", *American Economic Review*, 78(2), 96–103.

[3] Allen F. (1990), "The Market for Information and the Origin of Financial Intermediation", *Journal of Financial Intermediation*, 1, 3–30.

[4] Benabou, R. and Laroque, M. (1992), "Using privileged information to manipulate markets: insiders, gurus and credibility", *Quarterly Journal of Economics*, 921–958.

[5] Benveniste L.M., Marcus A.J., and Wilhelm W.J. (1992), "What's special about the specialist ?", *Journal of Financial Economics*, 32, 61–86.

[6] Bhattacharya and Pfleiderer (1985), "Delegated portfolio management", *Journal of Economic Theory*, **36**, 1–25.

[7] Chakravarty, S. and Sarkar, A. (2000), "A Model of Broker's trading with Applications To Order Flow Internalization", forthcoming in *Review of Financial Economics.*

[8] Coval, J. and Shumway, T. (1998), "Is sound just noise", Working paper, University of Michigan.

[9] Domowitz, I and Steil, B. (1999), "Automation, trading costs and the structure of the Securities Trading Industry", *Brookings-Wharton Papers on Financial Services*, **2**, 33-92.

[10] Fishman M. and Hagerty K. (1995), "The Incentive to Sell Financial Market Information", *Journal of Financial Intermediation*, **4**, 95–115.

[11] Fishman, M. and F. Longstaff (1992), "Dual Trading in Futures markets", *Journal of Finance*, **47**, 643–669.

[12] Griffiths, M., Smith, B., Turnbull, A. and R. White (1998), "Information Flows and Open Outcry: Evidence of Imitation Trading", *Journal of International Finnacial Markets, Institutions and Money*, **8**, 101–116.

[13] Harris (2000), L., “Floor versus automated trading systems: a survey of the issues”, LHarris.USC.edu/Acrobat/floor.

[14] Hasbrouck, J., Sofianos, G. and Sosebee, D. (1993), “NYSE Systems and Trading Procedures”, NYSE Working Paper, 93-01.

[15] Kofman, P. and Moser, J. (1997), “Spreads, information flows and transparency across trading systems”, *Applied Financial Economics*,**7**, 281–294.

[16] Kyle A. S. (1985), “Continuous Auctions and Insider Trading”, *Econometrica*, 6, 1315–1335.

[17] Madhavan, A. (1996), “Security prices and market transparency”, *Journal of Financial Intermediation*, **5**, 255–283.

[18] Madrigal V. (1996), “Non-Fundamental Speculation”, *Journal of Finance*, 51(2), 553–576.

[19] Pirrong, C. (1996), “Market liquidity and depth on computerized and open outcry trading systems: a comparison of DTB and LIFFE Bund contracts”, *Journal of Futures Markets*, **16**, 519–543.

[20] Röell A. (1990), “Dual-Capacity Trading and the Quality of the Market”, *Journal of Financial Intermediation*, 1, 105–124.

[21] Sarkar, A. (1995) “Dual trading: winners, losers and market impact”, *Journal of Financial Intermediation*, **4**, 77-93.

[22] Sarkar, A. and Wu, L. (1999) “Conflict of interests and execution quality of futures floor traders”, Working Paper, New-York FED.

[23] Shyy, G. and Lee, J.H., (1995) “Price transmission and information asymmetry in Bund futures markets: LIFFE vs. DTB”, *Journal of Futures Markets*, **15**, 87-99.

[24] Sofianos G., and Werner I. (1997), “The trades of NYSE Floor Brokers”, NYSE Working Paper 97-04.

[25] Theissen, E. (1999), “Floor versus Screen Trading: Evidence from the German Stock Market”, Working Paper CR 690, Groupe HEC.

[26] Vives X.(1995), "The Speed of Information Revelation in a Financial Market Mechanism", *Journal of Economic Theory*, 67, 178–204.

[27] Venkataraman, K. (2000) "Automated versus Floor Trading: An Analysis of execution Costs on the Paris and New-York Exchanges," Working Paper, Southern Methodist University.

6 Appendix

Proof of Lemma 1

Step1: The optimal trading strategy for speculator S. Let $y_S = (\tilde{v}, \hat{v}, \hat{x})$ be the information set of speculator S. The latter chooses his market order, Q^S, so as to maximize his expected profit

$$\pi^S(y_S) = E(Q^S(\tilde{v} - p(\tilde{O})) \mid y_S).$$

The first order condition yields

$$Q^S(y_S) = \frac{(\tilde{v} - \mu) - \lambda \times E\left[Q^B(y_B) + \sum_{j=2}^{j=N} Q^i(\tilde{v}) + \tilde{x}_0 + \tilde{x}_B \mid y_S\right]}{2\lambda}. \tag{9}$$

Notice that

$$E\left(Q^i(\tilde{v}) \mid y_S\right) = a'(\tilde{v} - \mu),$$

and

$$E\left(Q^B(y_B) \mid y_S\right) = b_1 E\left(\tilde{x}_B \mid \hat{x}\right) + b_2\hat{x} + b_3(\hat{v} - \mu),$$

and

$$E\left(\tilde{x}_B \mid \hat{x}\right) = \frac{\sigma_B^2 \hat{x}}{\sigma_B^2 + \sigma_\eta^2}.$$

Substituting these expressions in Equation (9) yields

$$\begin{aligned}
Q^S(y_S) &= \frac{(\tilde{v} - \mu)}{2\lambda} - \frac{1}{2} \times \left[(N-1)\, a'(\tilde{v} - \mu) + b_3(\hat{v} - \mu) + (b_1 + 1)\frac{\sigma_B^2}{\sigma_B^2 + \sigma_\eta^2}\hat{x} + b_2\hat{x}\right] \\
&= \left(\frac{1}{2\lambda} - \frac{(N-1)\, a'}{2}\right)(\tilde{v} - \mu) - \frac{b_3}{2}(\hat{v} - \mu) - \frac{1}{2}\left((b_1 + 1)\frac{\sigma_B^2}{\sigma_B^2 + \sigma_\eta^2} + b_2\right)\hat{x}.
\end{aligned}$$

Hence,

$$\begin{aligned}
a_1 &= \left(\frac{1}{2\lambda} - \frac{(N-1)\, a'}{2}\right) \\
a_2 &= -\frac{b_3}{2} \\
a_3 &= -\frac{1}{2}\left((b_1 + 1)\frac{\sigma_B^2}{\sigma_B^2 + \sigma_\eta^2} + b_2\right)
\end{aligned}$$

Step 2: The optimal trading strategy for speculator i, $i \neq S$.

Speculator i chooses his market order, Q^i, so as to maximize his expected profit

$$\pi^i(v) = E(Q^i(\tilde{v} - p(\tilde{O})) \mid \tilde{v} = v).$$

The first order condition yields

$$Q^i(\tilde{v}) = \frac{(\tilde{v} - \mu) - \lambda \cdot E\left[Q^S(y_S) + \sum_{j=2}^{j=N} Q^j(\tilde{v}) + Q^B(y_B) + \tilde{x} \mid \tilde{v} = v\right]}{\lambda} \tag{10}$$

We focus on symmetric trading strategies for all the speculators $i \neq S$. This imposes $Q^j(\tilde{v}) = Q^i(\tilde{v}), \quad \forall j \neq i$. Substituting Q_j by Q_i in Equation (10) yields

$$Q^i(\tilde{v}) = \frac{(\tilde{v} - \mu)}{N\lambda} - \frac{1}{N} \cdot \left[E\left[Q^S(y_S) \mid \tilde{v}\right] + E\left[Q^B(y_B) \mid \tilde{v} = v\right]\right]. \tag{11}$$

Furthermore

$$E\left(Q^S(y_S) \mid \tilde{v} = v\right) = (a_1 + a_2)(\tilde{v} - \mu),$$

and

$$E\left(Q^B(y_B) \mid \tilde{v} = v\right) = b_3(\tilde{v} - \mu).$$

Consequently

$$Q^i(\tilde{v}) = \left(\frac{1}{N\lambda} - \frac{(a_1 + a_2 + b_3)}{N}\right)(\tilde{v} - \mu). \tag{12}$$

We deduce that

$$a' = \frac{1}{N\lambda} - \frac{(a_1 + a_2 + b_3)}{N}. \tag{13}$$

Step 3: The optimal trading strategy for speculator B. We denote $y_B = (\tilde{x}_B, \hat{v}, \hat{x})$, the information set of speculator B. She chooses her market order, Q^B, so as to maximize

$$\pi^B(y_B) = E(Q^B(\tilde{v} - p(\tilde{O})) \mid y_B).$$

The first order condition yields

$$
\begin{aligned}
Q^B\left(y_B\right) &= \frac{E\left(\tilde{v} \mid \hat{v}\right) - \mu - \lambda E\left[Q^S(y_S) + \sum_{i=2} Q^i(\tilde{v}) + \sum x_i \mid y_B\right]}{2\lambda} \\
&= \frac{E\left(\tilde{v} \mid \hat{v}\right) - \mu - \lambda E\left[Q^S(y_S) + (N-1)\, Q^i(\tilde{v}) + \tilde{x} \mid y_B\right]}{2\lambda}.
\end{aligned}
$$

We notice that

$$
E\left(Q^S(y_S) \mid y_B\right) = a_1 E\left(\tilde{v} - \mu \mid \hat{v}\right) + a_2(\hat{v} - \mu) + a_3\hat{x},
$$

and

$$
E\left(Q^i(\tilde{v}) \mid y_B\right) = a' E\left(\tilde{v} - \mu \mid \hat{v}\right),
$$

and that

$$
E\left(\tilde{v} - \mu \mid \hat{v}\right) = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}(\hat{v} - \mu).
$$

Substituting these expressions in the first order condition for Speculator B yields (after some algebra)

$$
Q^B(y_B) = -\frac{\tilde{x}_B}{2} - \frac{a_3}{2}\hat{x} + \frac{1}{2\lambda}\left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} - \lambda a_2 - \lambda\left(a_1 + (N-1)\, a'\right)\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\right)(\hat{v} - \mu). \tag{14}
$$

Hence,

$$
\begin{aligned}
b_1 &= -\frac{1}{2}, \\
b_2 &= -\frac{a_3}{2}, \\
b_3 &= \frac{1}{2\lambda}\left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} - \lambda a_2 - \lambda\left(a_1 + (N-1)\, a'\right)\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\right).
\end{aligned}
$$

Steps 1 to 3 give us 9 equations with 9 unknowns (a_1, a_2 etc...). Solving this system of equations yield

$$
\begin{aligned}
a_1 &= \frac{3\left(\sigma_v^2+\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)},\\
a_2 &= -\frac{\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)}\\
a_3 &= -\frac{\sigma_B^2}{3\left(\sigma_B^2+\sigma_\eta^2\right)},\\
a' &= \frac{2\sigma_v^2+3\sigma_\varepsilon^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)},\\
b_1 &= -\frac{1}{2},\\
b_2 &= \frac{\sigma_B^2}{6\left(\sigma_B^2+\sigma_\eta^2\right)},\\
b_3 &= \frac{2\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)},
\end{aligned}
$$

Step 4. Computation of λ. Recall that

$$p(O)=E(\tilde{v}\mid\tilde{O}=O).$$

Given speculators' trading rules,

$$O=Q^S\left(y_S\right)+\left(N-1\right)Q^i(\tilde{v})+Q^B\left(y_B\right)+\tilde{x}$$

$$=\left(a_1+\left(N-1\right)a'\right)\left(\tilde{v}-\mu\right)+\left(a_2+b_3\right)\left(\hat{v}-\mu\right)+\left(a_3+b_2\right)\hat{x}+\left(b_1+1\right)\tilde{x}_B+\tilde{x}_0.$$

Hence $\tilde{O}$ is normally distributed, with mean zero. Consequently

$$p(O)=\mu+\lambda O,$$

with

$$\lambda=\frac{Cov(\tilde{v},\tilde{O})}{Var(\tilde{O})}. \tag{15}$$

Now

$$cov\left(\tilde{v},\tilde{O}\right)=\left(a_1+\left(N-1\right)a'+a_2+b_3\right)\sigma_v^2=\frac{\left(2\sigma_v^2\left(N+1\right)+3N\sigma_\varepsilon^2\right)\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)}, \tag{16}$$

and

$$
\begin{aligned}
Var\left(O\right) &= \left(a_1 + \left(N - 1\right)a'\right)^2 \sigma_v^2 + \left(a_2 + b_3\right)^2 \left(\sigma_v^2 + \sigma_\varepsilon^2\right) + 2\left(a_1 + \left(N - 1\right)a'\right)\left(a_2 + b_3\right)\sigma_v^2 \\
&\quad + \left(a_3 + b_2\right)^2 \left(\sigma_B^2 + \sigma_\eta^2\right) + \left(b_1 + 1\right)^2 \sigma_B^2 + 2\left(a_3 + b_2\right)\left(b_1 + 1\right)\sigma_B^2 + \sigma_0^2 \\
&= \left(a_1 + \left(N - 1\right)a' + a_2 + b_3\right)^2 \sigma_v^2 + \left(\frac{b_3}{2}\right)^2 \sigma_\varepsilon^2 + \left(\frac{a_3}{2} + \frac{1}{2}\right)^2 \sigma_B^2 + \left(\frac{a_3}{2}\right)^2 \sigma_\eta^2 + \sigma_0^2 \\
&= \sigma_v^2 \frac{\left(2\sigma_v^2\left(N+1\right) + 3N\sigma_\varepsilon^2\right)^2 + \sigma_v^2\sigma_\varepsilon^2}{\left(\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)\right)^2} - \frac{5\sigma_B^4}{36\left(\sigma_B^2 + \sigma_\eta^2\right)} + \frac{\sigma_B^2}{4} + \sigma_0^2.
\end{aligned}
$$

We deduce that

$$
\lambda = \frac{6\sqrt{\sigma_v^2\left(\sigma_B^2 + \sigma_\eta^2\right)\left(4\left(N+1\right)\sigma_v^4 + \left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2 + 9N\sigma_\varepsilon^4\right)}}{\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)\sqrt{\sigma_B^2\left(4\sigma_B^2 + 9\sigma_\eta^2\right) + 36\sigma_0^2\left(\sigma_B^2 + \sigma_\eta^2\right)}}. \blacksquare \qquad (17)
$$

Proof of Lemma 2

We write the equilibrium value of λ in the following way:

$$
\begin{aligned}
\lambda\left(\sigma_\varepsilon^2, \sigma_\eta^2\right) &= 6\frac{\sqrt{\sigma_v^2\left(4\left(N+1\right)\sigma_v^4 + \left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2 + 9N\sigma_\varepsilon^4\right)}}{\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)} \times \frac{\sqrt{\sigma_B^2 + \sigma_\eta^2}}{\sqrt{\sigma_B^2\left(4\sigma_B^2 + 9\sigma_\eta^2\right) + 36\sigma_0^2\left(\sigma_B^2 + \sigma_\eta^2\right)}} \\
&= 6 \times \lambda_1\left(\sigma_\varepsilon^2\right) \times \lambda_2\left(\sigma_\eta^2\right).
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\frac{\partial\lambda\left(\sigma_\varepsilon^2, \sigma_\eta^2\right)}{\partial\sigma_\eta^2} &= 6 \times \lambda_1\left(\sigma_\varepsilon^2\right) \times \frac{\partial\lambda_2\left(\sigma_\eta^2\right)}{\partial\sigma_\eta^2} \\
&= -\frac{15 \times \lambda_1\left(\sigma_\varepsilon^2\right) \times \left(\sigma_B^2\right)^2}{\sqrt{\sigma_B^2 + \sigma_\eta^2}\left(\sigma_B^2\left(4\sigma_B^2 + 9\sigma_\eta^2\right) + 36\sigma_0^2\left(\sigma_B^2 + \sigma_\eta^2\right)\right)^{\frac{3}{2}}} < 0, \\
\frac{\partial\lambda\left(\sigma_\varepsilon^2, \sigma_\eta^2\right)}{\partial\sigma_\varepsilon^2} &= 6 \times \lambda_2\left(\sigma_\eta^2\right) \times \frac{\partial\lambda_1\left(\sigma_\varepsilon^2\right)}{\partial\sigma_\varepsilon^2} \\
&= \frac{6 \times \lambda_2\left(\sigma_\eta^2\right) \times \sigma_v^4\left(3\left(7N - 5\right)\sigma_\varepsilon^2 + 2\left(5N - 2\right)\sigma_v^2\right)}{2\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)^2\sqrt{\sigma_v^2\left(4\left(N+1\right)\sigma_v^4 + \left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2 + 9N\sigma_\varepsilon^4\right)}} > 0.
\end{aligned}
$$

We also observe that

$$\begin{aligned}
\lim_{\sigma_\varepsilon^2\to\infty} \lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right) &= 6\times\lambda_2\left(\sigma_\eta^2\right)\times\frac{\sqrt{N\sigma_v^2}}{(N+1)},\\
\lim_{\sigma_\eta^2\to\infty} \lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right) &= 2\times\lambda_1\left(\sigma_\varepsilon^2\right)\times\frac{1}{\sqrt{\sigma_B^2+4\sigma_0^2}}.
\end{aligned}$$

Consequently,

$$\lambda(\infty,\infty)=\lim_{\substack{\sigma_\eta^2\to\infty\\ \sigma_\varepsilon^2\to\infty}}\lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right)=6\times\frac{1}{\sqrt{9\sigma_B^2+36\sigma_0^2}}\times\lim_{\sigma_\varepsilon^2\to\infty}\lambda_1\left(\sigma_\varepsilon^2\right)=\frac{2\sqrt{N\sigma_v^2}}{(N+1)\sqrt{\sigma_B^2+4\sigma_0^2}}.\blacksquare \tag{18}$$

Proof of Lemma 3

We denote by $\pi^j(y_j)$, speculator j's expected profit given his information set y_j prior to trading at date 1 and by $\Pi^j(\sigma_\eta^2,\sigma_\varepsilon^2,N)$, his ex-ante expected profit, that is before observing information. Notice that

$$\pi^j(y_j)=Q^j\times E(\tilde{v}-\mu-\lambda\tilde{x}-\lambda Q^{-j}-\lambda Q^j\,|y_j). \tag{19}$$

The first order condition for speculator j imposes (see the proof of Lemma 1) that

$$2\lambda Q^j=E(\tilde{v}-\mu-\lambda\tilde{x}-\lambda Q^{-j}\,|y_j). \tag{20}$$

Hence, we deduce from Equations (19) and (20) that $\pi^j(y_i)=\lambda(Q^j)^2$. It follows that

$$\Pi^j=E(\pi^j(y_j))=\lambda\times Var(Q^j).$$

This implies that

$$\Pi^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)=\lambda\left(a_1^2Var\tilde{v}+a_3^2Var\hat{x}_1+a_2^2Var\hat{v}+2a_1a_2cov\left(\tilde{v},\hat{v}\right)\right),$$

which yield (using the expressions for a_1, a_2 and a_3)

$$\Pi^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)=\left(\frac{\sigma_v^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(4\sigma_v^2+9\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)^2}+\frac{\lambda\sigma_B^4}{9\left(\sigma_B^2+\sigma_\eta^2\right)}\right).$$

We define

$$\Pi_{nf}^{S} \stackrel{def}{=} \frac{\lambda\sigma_B^4}{9\left(\sigma_B^2+\sigma_\eta^2\right)},$$

and

$$\Pi_{f}^{S} \stackrel{def}{=} \left(\frac{\sigma_v^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(4\sigma_v^2+9\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)^2}\right).$$

We proceed exactly in the same way for speculator B.■

Proof of Proposition 2

The following lemma is useful for the proof.

Lemma 4 *: In absence of information sharing, speculator S has a larger expected profit than speculator B ($\Pi^B(\infty,\infty,N) \leq \Pi^S(\infty,\infty,N)$) iff*

$$\sigma_0^2 \geq \frac{(N-1)}{4+(N-1)}.$$

Proof: We have

$$\Pi^B(\infty,\infty,N) = \frac{\lambda(\infty,\infty)\sigma_B^2}{4}, \tag{21}$$

and

$$\Pi^S(\infty,\infty,N) = \frac{1}{\lambda(\infty,\infty)(N+1)^2}. \tag{22}$$

Using Equation (18) (proof of Lemma 2) we obtain that $\Pi^B(\infty,\infty,N) \leq \Pi^S(\infty,\infty,N)$ iff

$$\sigma_0^2 > \frac{(N-1)\sigma_B^2}{4}.$$

Then the result follows from the fact that $\sigma_B^2 = 1-\sigma_0^2$.■

When there is perfect information sharing, speculators B and S have the same expected profits given by

$$\Pi^S(0,0,N) = \Pi^B(0,0,N) = \frac{1}{\lambda(0,0)(N+2)^2} + \frac{\lambda(0,0)\sigma_B^2}{9}. \tag{23}$$

It follows that perfect information sharing is possible iff

$$\frac{1}{\lambda(0,0)(N+2)^2} + \frac{\lambda(0,0)\sigma_B^2}{9} \geq Max\{\Pi^B(\infty,\infty,N), \Pi^S(\infty,\infty,N)\}. \tag{24}$$

Case 1. $\sigma_0^2 \geq \frac{(N-1)}{4+(N-1)}$. In this case, using Lemma 4, we can rewrite Condition (24) as

$$\frac{1}{\lambda(0,0)(N+2)^2} + \frac{\lambda(0,0)\sigma_B^2}{9} \geq \Pi^S(\infty,\infty,N),$$

which yields (using Equation (22)),

$$\frac{1}{\lambda(0,0)(N+2)^2} + \frac{\lambda(0,0)\sigma_B^2}{9} \geq \frac{1}{\lambda(\infty,\infty)(N+1)^2} \tag{25}$$

It follows from the expression of λ (in the proof of Lemma 2) that

$$\lambda(0,0) = \frac{3\sqrt{N+1}}{(N+2)\sqrt{\sigma_B^2 + 9\sigma_0^2}},$$

and $\lambda(\infty,\infty)$ is given by equation (18). Using these expressions and the fact that $\sigma_B^2 = 1 - \sigma_0^2$, we rewrite (after some algebra) Equation (25) as

$$\left(\frac{(N+1)}{(N+2)}\right) G(N,\sigma_0^2) \geq 0,$$

with

$$G(N,\sigma_0^2) = \left[\frac{N+1}{N+2} + \frac{(N+1)^2(1-\sigma_0^2)}{(N+2)(1+8\sigma_0^2)} - \frac{3\sqrt{(N+1)(1+3\sigma_0^2)}}{2\sqrt{N(1+8\sigma_0^2)}}\right].$$

Notice that $G(N,.)$ decreases with σ_0^2. Furthermore G is strictly positive for $\sigma_0^2 = \frac{(N-1)}{4+(N-1)}$ and negative for $\sigma_0^2 = 1$. We conclude that there exists a cutoff $\sigma_0^{*2}(N) \in (\frac{(N-1)}{4+(N-1)}, 1)$ such that Condition (25) is satisfied iff $\sigma_0^2 \leq \sigma_0^{*2}(N)$. This cutoff is implicitly defined as the solution of

$$G(N,\sigma_0^2) = 0. \tag{26}$$

As $G(.,.)$ increases with N and decreases with σ_0^2, we deduce that $\sigma_0^{2*}(N)$ increases with N.

Case 2. $\sigma_0^2 < \frac{(N-1)}{4+(N-1)}$. In this case, using Lemma 4, we can rewrite Condition (24) as

$$\frac{1}{\lambda(0,0)(N+2)^2} + \frac{\lambda(0,0)\sigma_B^2}{9} \geq \Pi^B(\infty, \infty, N),$$

which yields (using Equation (21)),

$$\frac{1}{\lambda(0,0)(N+2)^2} + \frac{\lambda(0,0)\sigma_B^2}{9} \geq \frac{\lambda(\infty, \infty)\sigma_B^2}{4} \tag{27}$$

Using the expressions for $\lambda(0,0)$ and $\lambda(\infty, \infty)$, after some manipulations, we rewrite the previous condition as

$$F(N, \sigma_0^2) \stackrel{def}{=} \left(\frac{3(N+2)}{2\sqrt{(N+1)(1+3\sigma_0^2)}} \sqrt{N(1+8\sigma_0^2)} - (N+1) \right) \frac{(1-\sigma_0^2)}{1+8\sigma_0^2} - 1 \leq 0.$$

We observe that $F(N,.)$ decreases with σ_0^2. Furthermore $F > 0$ for $\sigma_0^2 = 0$ and $F < 0$ for $\sigma_0^2 = \frac{(N-1)}{4+(N-1)}$. It follows that there exists a cutoff $\underline{\sigma}_0^2(N) \in (0, \frac{(N-1)}{4+(N-1)})$ such that for $\sigma_0^2 \geq \underline{\sigma}_0^2(N)$, Condition (24) is satisfied. This cutoff is implicitly defined as the solution of

$$F(N, \sigma_0^2) = 0.$$

As $F(.,.)$ increases with N and decreases with σ_0^2, we deduce that $\underline{\sigma}_0^2(N)$ increases with N. Furthermore we have

$$0 < \underline{\sigma}_0^2(N) < \frac{(N-1)}{4+(N-1)} < \sigma_0^{2*}(N) < 1. \blacksquare$$

Proof of Proposition 3

Step 1: Prices are more informative when there is information sharing. Recall that $\tilde{v}$ and $\tilde{p}$ are normally distributed and that $\tilde{p}(O) = \mu + \lambda O$. Therefore

$$Var(\tilde{v} \mid \tilde{p}(O) = p) = \sigma_v^2 - \frac{Cov^2(\tilde{v}, \tilde{O})}{Var(\tilde{O})}.$$

Using Equations (15) and (16) which appear in the proof of Lemma 1, we obtain that

$$Var(\tilde{v} \mid \tilde{p}(O) = p) = \sigma_v^2 - \lambda Cov(\tilde{v}, \tilde{O}) = \sigma_v^2 - \frac{(2\sigma_v^2 (N+1) + 3N\sigma_\varepsilon^2)\, \sigma_v^2}{(2\,(N+2)\,\sigma_v^2 + 3\,(N+1)\,\sigma_\varepsilon^2)}.$$

It is immediate that $Var(\tilde{v} \mid \tilde{p}(O) = p)$ increases with σ_ε^2 and does not depend on σ_η^2. This means that information sharing (a decrease in σ_ε^2 and σ_η^2) makes equilibrium prices more informative.

Step 2: Prices are less volatile when there is information sharing.

Observe that

$$Var(\tilde{v} - p) = E(E((\tilde{v} - p)^2 \mid \tilde{p} = p)).$$

As $\tilde{p} = E(\tilde{v} \mid \tilde{p})$, the previous equality implies that

$$Var(\tilde{v} - p) = E(Var(\tilde{v} \mid \tilde{p} = p)).$$

Finally since $\tilde{v}$ and $\tilde{p}$ are normally distributed, $Var(\tilde{v} \mid \tilde{p} = p)$ is constant so that

$$Var(\tilde{v} - p) = Var(\tilde{v} \mid \tilde{p} = p).$$

Hence prices are less volatile when there is information sharing since prices are more informative in this case.■

Proof of Proposition 4

Consider the following ratio

$$H(N, \sigma_0^2) = \frac{\lambda(0,0)}{\lambda(\infty, \infty)}.$$

Perfect information sharing improves market liquidity if and only if

$$H(N, \sigma_0^2) < 1.$$

Using the expression for λ given in the proof of Lemma 2, we obtain

$$H(N, \sigma_0^2) = \frac{3(N+1)\sqrt{(N+1)(1+3\sigma_0^2)}}{2(N+2)\sqrt{N(1+8\sigma_0^2)}}.$$

It is immediate that $H(N,.)$ decreases with σ_0^2. Furthermore $H(N,1) < 1$ and $H(N,0) > 1$. Therefore there exists a threshold $\bar{\sigma}_0^2(N)$ such that $H < 1$ iff $\sigma_0^2 > \bar{\sigma}_0^2(N)$. This threshold solves

$$H(N, \sigma_0^2) = 1.$$

Solving this equation, we deduce that

$$\bar{\sigma}_0^2(N) = \frac{1 - h^2(N)}{8h^2(N) - 3},$$

where $h(N) = \frac{2(N+2)\sqrt{N}}{3(N+1)\sqrt{N+1)}} < 1$. As $h(N) \geq 2/3$, we have $\bar{\sigma}_0^2 < 1$. ∎

Proof of Proposition 5

The expected trading costs for the liquidity traders when there is information sharing are

$$E\left(CT^e\right) = \lambda \left(\frac{6\sigma_0^2\left(\sigma_B^2 + \sigma_\eta^2\right) + \left(2\sigma_B^2 + 3\sigma_\eta^2\right)\sigma_B^2}{6\left(\sigma_B^2 + \sigma_\eta^2\right)} \right).$$

Using the expression for λ, we rewrite this equation as

$$E\left(CT^e\right) = \frac{\sqrt{\sigma_v^2\left(4\left(N+1\right)\sigma_v^4 + \left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2 + 9N\sigma_\varepsilon^4\right)}\left(6\sigma_0^2\left(\sigma_B^2 + \sigma_\eta^2\right) + \left(2\sigma_B^2 + 3\sigma_\eta^2\right)\sigma_B^2\right)}{\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)\sqrt{\left(\sigma_B^2 + \sigma_\eta^2\right)\left[\sigma_B^2\left(4\sigma_B^2 + 9\sigma_\eta^2\right) + 36\sigma_0^2\left(\sigma_B^2 + \sigma_\eta^2\right)\right]}}.$$

When the brokers do not share their information, then

$$\begin{aligned} E\left(CT^{ne}\right) &= E\left[\left(P\left(O\right) - \tilde{v}\right) \times \tilde{x}\right] = \lambda^{ne}\left(\sigma_0^2 + \frac{1}{2}\sigma_B^2\right) \\ &= \frac{\sqrt{\sigma_v^2 N}}{\left(N+1\right)} \times \frac{\left(2\sigma_0^2 + \sigma_B^2\right)}{\sqrt{\sigma_B^2 + 4\sigma_0^2}}. \end{aligned}$$

We denote Φ the difference between the expected trading costs when there is information sharing and when there is no information sharing. Hence

$$\Phi\left(N, \sigma_\varepsilon^2, \sigma_\eta^2\right) = E\left(CT^e\right) - E\left(CT^{ne}\right)$$

Straightforward manipulations show that

$$\frac{\sqrt{\sigma_v^2\left(4\left(N+1\right)\sigma_v^4+\left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2+9N\sigma_\varepsilon^4\right)}}{\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)}<\frac{\sqrt{\sigma_v^2 N}}{\left(N+1\right)} \tag{28}$$

Now consider the following function

$$\psi\left(\sigma_\eta^2\right)=\frac{\left(6\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)+\left(2\sigma_B^2+3\sigma_\eta^2\right)\sigma_B^2\right)^2}{\left(\sigma_B^2+\sigma_\eta^2\right)\left[\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right]}-\frac{\left(2\sigma_0^2+\sigma_B^2\right)^2}{\sigma_B^2+4\sigma_0^2}.$$

As $\sigma_0^2=1-\sigma_B^2$, we rewrite the previous equation as

$$\psi\left(\sigma_\eta^2\right)=\frac{\left(6\left(\sigma_B^2+\sigma_\eta^2\right)-\sigma_B^2\left(4\sigma_B^2+3\sigma_\eta^2\right)\right)^2}{\left(\sigma_B^2+\sigma_\eta^2\right)\left[36\left(\sigma_B^2+\sigma_\eta^2\right)-\sigma_B^2\left(32\sigma_B^2+27\sigma_\eta^2\right)\right]}-\frac{\left(2-\sigma_B^2\right)^2}{4-3\sigma_B^2}.$$

Observe that

$$\psi\left(0\right)=\frac{\sigma_B^2\left(-7+11\sigma_B^2-4\sigma_B^4\right)}{\left(9-8\sigma_B^2\right)\left(4-3\sigma_B^2\right)}<0,\text{ since }\sigma_B^2\in\left[0,1\right]$$

and

$$\lim_{\sigma_\eta^2\to\infty}\psi\left(\sigma_\eta^2\right)=0.$$

and

$$\psi'\left(\sigma_\eta^2\right)=\frac{\sigma_B^4\left(176\sigma_B^8+144\sigma_B^6\left(2\sigma_\eta^2-3\right)-72\sigma_B^2\sigma_\eta^2\left(5\sigma_\eta^2-7\right)+9\sigma_B^4\left(13\sigma_\eta^4-88\sigma_\eta^2+28\right)+252\sigma_\eta^4\right)}{\left(\sigma_B^2+\sigma_\eta^2\right)^2\left[36\left(\sigma_B^2+\sigma_\eta^2\right)-\sigma_B^2\left(32\sigma_B^2+27\sigma_\eta^2\right)\right]^2} \tag{29}$$

Now we remark that if $\sigma_B^2\in\left[0,\frac{21}{22}\right]$, then $\psi'\left(\sigma_\eta^2\right)>0$ and therefore $\psi\left(\sigma_\eta^2\right)<0$. If $\sigma_B^2\in\left[\frac{21}{22},1\right]$, then there is a unique value of σ_η^2 such that $\psi'=0$. This value is

$$\bar{\sigma}_\eta^2=\frac{2\sigma_B^2\left(22\sigma_B^2-21\right)}{3\left(14-13\sigma_B^2\right)}.$$

Hence ψ has only one extremum and this extremum is a minimum since

$$\psi''\left(\bar{\sigma}_\eta^2\right) = \frac{27\left(14 - 13\sigma_B^2\right)^4}{625\sigma_B^8\left(2\sigma_B^2 - 1\right)} > 0,$$

We deduce that $\forall\sigma_\eta^2$ and $\forall\sigma_B^2$, $\psi\left(\sigma_\eta^2\right) < 0$. We conclude that

$$\sqrt{\frac{6\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)+\left(2\sigma_B^2+3\sigma_\eta^2\right)\sigma_B^2}{\left(\sigma_B^2+\sigma_\eta^2\right)\left[\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right]}} < \sqrt{\frac{\left(2\sigma_0^2+\sigma_B^2\right)}{\sigma_B^2+4\sigma_0^2}}. \tag{30}$$

Using Inequality (28) and Inequality (30), we deduce that $\Phi\left(N,\sigma_\varepsilon^2,\sigma_\eta^2\right) < 0$ which means that the expected trading costs are always lower when there is information sharing. $\blacksquare$

FIGURE 1: Is Perfect Information Sharing Possible?

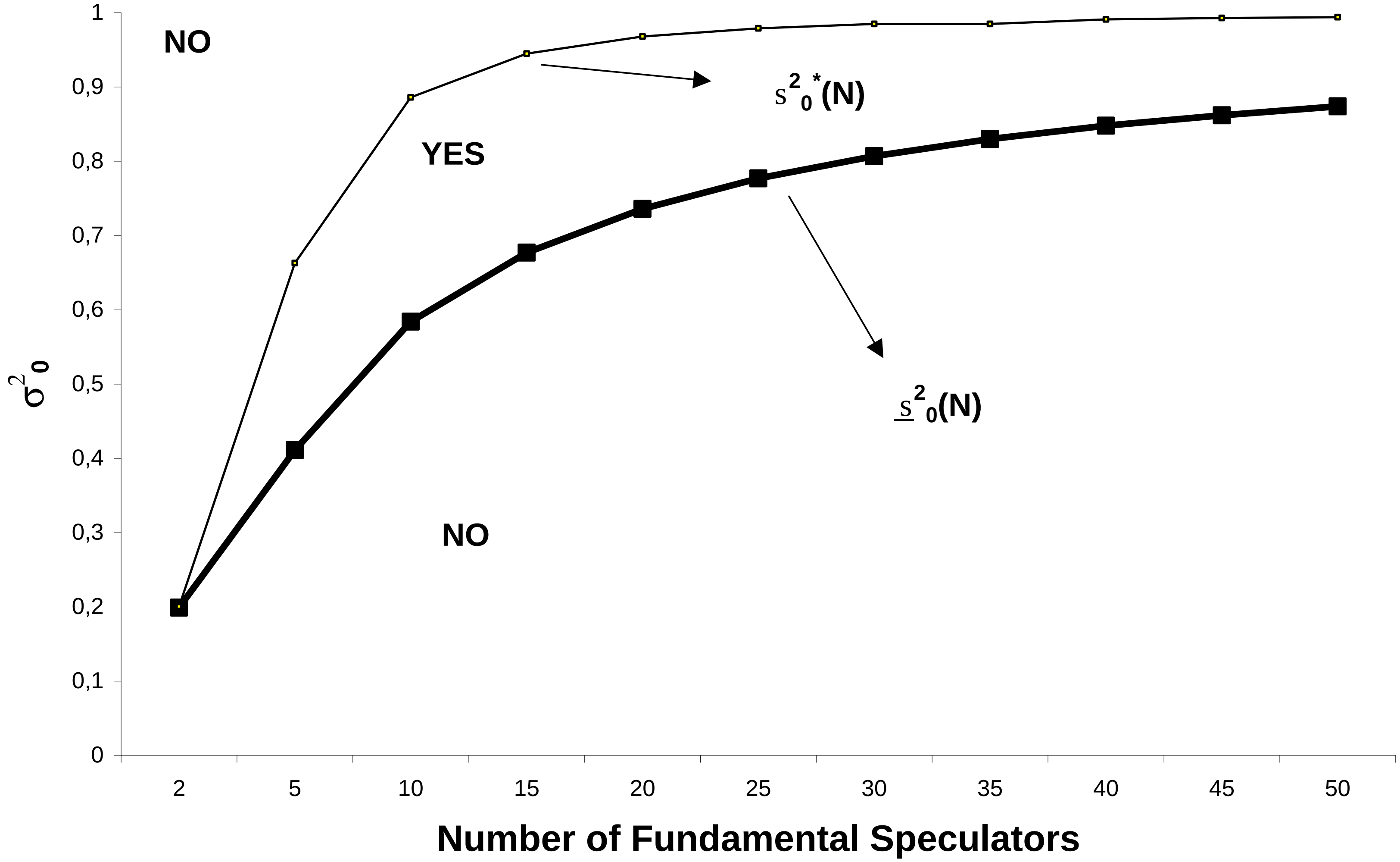

FIGURE 2: Does Perfect Information Sharing Improve Liquidity?